CHAROTAR ENGLISH MEDIUM SCHOOL

Std : 9 | Second Terminal Exam | Marks: 50

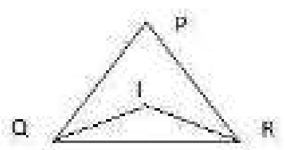
Date : 28 -01-19 | Sub: Mathematics | Time: 2 hours

Answer Key

Section A

- Do as directed (Q. No. 1 to 10) (Each 1 mark) [10]

1) In ΔABC, ∠A = ∠C , AC = 5 and BC = 4 . Then, the perimeter of ΔABC is …… .
   .Given triangle is isosceles triangle ; ∠A = ∠C so AB = BC ,
   AB = 4 Hence perimeter of ΔABC = AB+BC+AC = 4+4+5 = 13
2) In ΔPQR , the bisector of ∠Q and ∠R intersect at I . If ∠ P = 70° , then find m∠ QIR .



m∠ QIR = 180 - (m∠IQR + m∠IRQ)

m∠ QIR = 180 – 55 (m∠IQR & m∠IRQ are bisector of ∠Q and ∠R)

m∠ QIR = 125°

3) ABCD is a rhombus such that ∠ACB = 40° . What will be ∠ADB ?
   Given ABCD is a rhombus. Diagonals bisect each other perpendicularly.
   Hence ∠BOC = 90°
   Given ∠OCB = 40°
   AD||BC and BD is the transversal
   ∴ ∠ADB = ∠DBC (Alternate angles)
   Hence in right angled DBOC,
   ∠BOC + ∠OCB + ∠OBC = 180°
   ⇒ 90° + 40° + ∠OBC = 180°
   ⇒130° + ∠OBC = 180°
   ⇒ ∠OBC = 180° - 130° = 50°
   But ∠OBC = ∠DBC
   Therefore, ∠ADB = 50
4) Simplify : $13^{1/5} . 17^{1/5} = (13.17)^{1/5+1/5} = (221)^{1/5}$
5) ABCD is a square . If area of ABCD = 36 cm^2 then AB = …. cm.
   Ar □ABCD = $(side)^2$ = AB x CD

$(36)^2 = AB^2$ = > **AB = 6 cm**

6) State the Property of a median of a triangle.

Any median of a triangle divides the triangle into two triangles of equal area.

7) If a triangle and a parallelogram are on the same base and between same parallels, then what is the ratio of the area of the triangle to the area of parallelogram?

Area of a triangle is half the area of parallelogram if they are standing on the same base and between same parallels. ***Hence, in such a case, the ratio of the area of the triangle to that of the parallelogram = 1 : 2.***

8) How many minimum measurement are required to construct a triangle? List out it.

There are three rules for congruency. All the three specify three data. They are :

1. Length of the three sides
2. Length of 2 sides and included angle
3. Two angles and the length of the included line (between the two angles)

Write down true or false for following sentences:

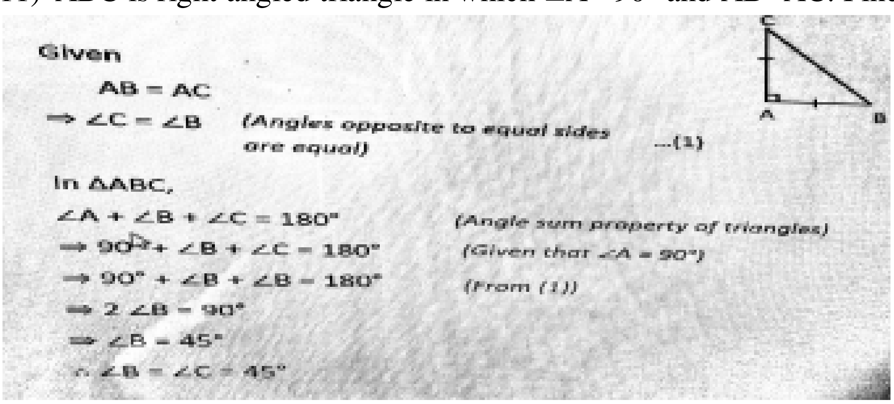
9) Congruent triangles are similar. **TRUE**

10) Axiom has proof. **FALSE**

Section B

- Answer the following questions in short with calculation : (Each 2 mark) [16]

11) ABC is right angled triangle in which ∠A =90° and AB=AC. Find $\angle B$ and $\angle C$.



Given

AB = AC

⇒ ∠C = ∠B (Angles opposite to equal sides are equal) ---(1)

In ΔABC,

∠A + ∠B + ∠C = 180° (Angle sum property of triangles)

⇒ 90° + ∠B + ∠C = 180° (Given that ∠A = 90°)

⇒ 90° + ∠B + ∠B = 180° (From (1))

⇒ 2 ∠B = 90°

⇒ ∠B = 45°

∴ ∠B = ∠C = 45°

12) State midpoint theorems.

The line segment joining the midpoints of two sides of a triangle is parallel to the third side .

The line drawn through the midpoint of one side of a triangle , parallel to another side bisects the third side.

13) Find the value of k, if x-1 is a factor of $4x^3+3x^2-4x+k$.

X - 1 is a factor of $4x^3 + 3x^2 - 4x + k$

then x=1 is one root of $4x^3 + 3x^2 - 4x + k$

put x= 1;

$P(1) = 4x^3 + 3x^2 - 4x + k$

$0 = 4x^3 + 3x^2 - 4x + k$

$0 = 4(1)^3 + 3(1)^2 - 4(1) + k$

$4 + 3 - 4 + k = 0$

$k = -3$

OR

Evaluate this with suitable identities: $(102)^3$

$(102)^3 = (100 + 2)^3$

Using $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Putting $a = 100$ & $b = 2$

$= (100)^3 + (2)^3 + 3(100)(2)(100 + 2)$

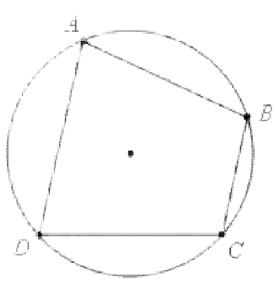
$= 1000000 + 8 + 600(102)$

$= 1000000 + 8 + 61200$

$= 1061208$

14) Define cyclic quadrilateral. Draw cyclic Quadrilateral ABCD.

A quadrilateral is defined as cyclic quadrilateral if all the four verticies of it lie on a circle.



15) In cyclic quadrilateral ABCD, $\angle A - \angle C = 20°$. Then find $m\angle A$

In cyclic quadrilateral sum of opposite angles are 180.

$m\angle A + m\angle C = 180°$

$\underline{m\angle A - m\angle C = 20°}$

$2\ m\angle A = 200$

$m\angle A = 100$

16) Simplify: $(\sqrt{3} + \sqrt{7})^2$

$(\sqrt{3}+\sqrt{7})^2$

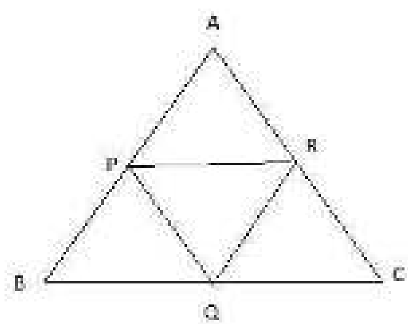
We know that, $[(a+b)^2=a^2+2ab+b^2]$

$=(\sqrt{3})^2+2(\sqrt{3})(\sqrt{7})+(\sqrt{7})^2$

$=3+2\sqrt{21}+7$

$=10+2\sqrt{21}$

17) In ΔABC, P, Q and R are the midpoints of AB, BC, and CA respectively. If area (ABC) = $40 cm^2$, then find area of (PBCR).



Area of $\Delta ABC = 40\ cm^2$

area of Δ APR= area of Δ PQR = area of Δ PBQ = area of Δ QRC = $\frac{1}{4}$ area of Δ ABC = $10\ cm^2$

Area of (PBCR) = area of Δ PBQ + area of Δ PQR + area of Δ QRC

= 10+10+10 = **$30\ cm^2$**

18) In □ ABCD, the measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are in proportion 2:4:5:4 . Find the measures of each angle.

In □ ABCD,

$m\angle A + m\angle B + m\angle C + m\angle D = 360$

$2x+4x+5x+4x = 360$

$15x = 360$

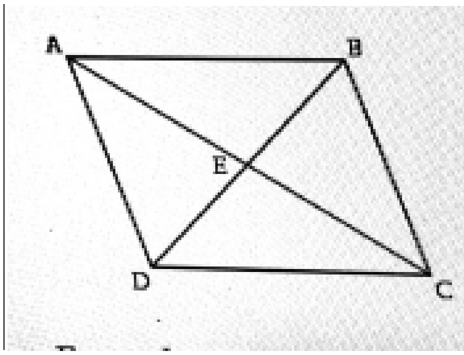
$X = 360/15$

$X = 24$

$m\angle A = 48$; $m\angle B = 96$; $m\angle C = 120$; $m\angle D = 96$

OR

18) ABCD is a Rhombus . If AC = 10cm and BD = 24cm , then find the perimeter of ABCD.



AC= 24 cm and BD=10 cm

AE=EC=12 cm and BE=ED=5 cm

Now, in triangle AED, By Pythagoras Theorem

$AD^2 = AE^2 + ED^2$

$AD^2 = 12^2 + 5^2$

$AD^2 = 144 + 25$

$AD^2 = 169$

AD = 13 cm

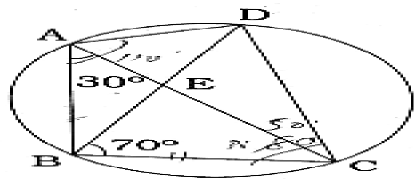
Perimeter = AB+BC+CD+DA = 13+13+13+13= 52cm

Section C

- Calculate and give the answers of following questions. (Each 3marks) [12]

19) ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70^\circ$, $\angle BAC = 30^\circ$, Then find $\angle BCD$. And if AB=BC, find $\angle ECD$.

ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If $\angle DBC = 70°$, $\angle BAC$ is $30°$, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$.



$\angle DAC = \angle DBC$ (Theorem 10.9)

$\therefore \angle DAC = 70°$ (angle sub in same segment).

$\angle BAD = \angle BAC + \angle DAC$ (Adjacent angles)

$\therefore \angle BAD = 30° + 70°$

$\therefore \angle BAD = 100°$

In cyclic quadrilateral ABCD,

$\angle BAD + \angle BCD = 180°$ (Theorem 10.11)

$\therefore 100° + \angle BCD = 180°$

$\therefore$ **$\angle BCD = 80°$**

In ΔABC, if AB = BC, then $\angle BAC = \angle BCA$

$\therefore 30° = \angle BCA$

$\therefore \angle BCA = 30°$

$\angle BCD = \angle BCA + \angle ACD$ (Adjacent angles)

$\therefore 80° = 30° + \angle ACD$

$\therefore \angle ACD = 50°$

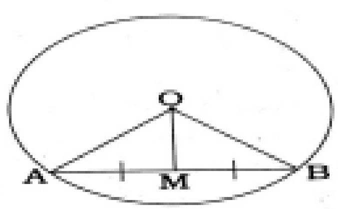
$\therefore$ **$\angle ECD = 50°$**

20) Prove that the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Theorem 10.4 : The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Given : AB is a chord of a circle with centre O. A line through O bisects AB at M, i.e., MA = MB.

To prove : OM $\perp$ AB.



Proof : In Δ OMA and Δ OMB,

OA = OB (Radii of the same circle)

OM = OM (Common)

MA = MB (Given)

$\therefore \Delta OMA \cong \Delta OMB$ (SSS congruence rule)

$\therefore \angle OMA = \angle OMB$ (CPCT)

OR

Factories: $4x^2+9y^2+16z^2+12xy-24yz-16xz$

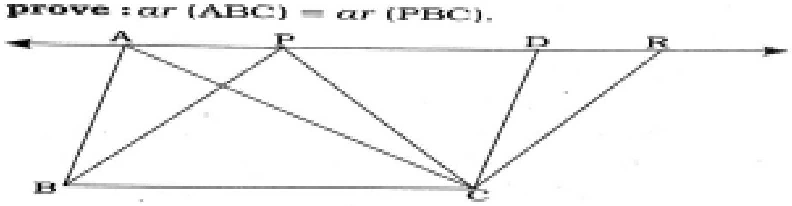
$$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$$
$$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$$
$$= [2x + 3y + (-4z)]^2$$
$$= (2x + 3y - 4z)^2$$
$$= \mathbf{(2x + 3y - 4z)(2x + 3y - 4z)}$$

21) Prove that two triangles on the same base and between the same parallels are equal in area.

Theorem 9.2 : Two triangles on the same base (or equal bases) and between the same parallels are equal in area.

Given : Δ ABC and Δ PBC are on the same base BC and between the parallels AP and BC.

To prove : ar (ABC) = ar (PBC).



Proof : Take points D and R on line AP such that CD ∥ BA and CR ∥ BP.

Then, we get parallelograms ABCD and PBCR which are on the same base BC and between the parallels BC and AR.

∴ ar (ABCD) = ar (PBCR) (Theorem 9.1)

∴ $\frac{1}{2} ar$ (ABCD) = $\frac{1}{2} ar$ (PBCR) ... ... (1)

Any diagonal of a parallelogram divides it into two congruent triangles.

Hence, ar (ABC) = $\frac{1}{2} ar$ (ABCD) and

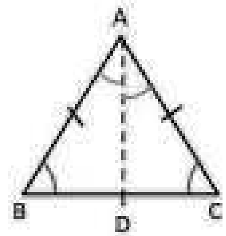
ar (PBC) = $\frac{1}{2} ar$ (PBCR) ... ... (2)

From (1) and (2),

ar (ABC) = ar (PBC)

OR

Angle opposite to equal sides of an isosceles triangle are equal.



Given :- Isosceles triangle ABC

i.e. AB = AC

To Prove :- ∠B = ∠C

Construction:- Draw a bisector of ∠A intersecting BC at D.

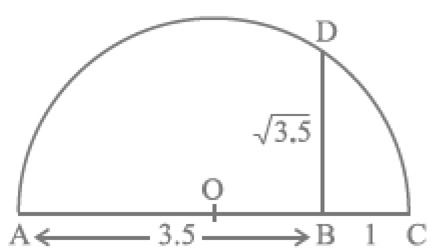
Proof:-

In △BAD and △CAD

AB = AC	(Given)
∠BAD = ∠CAD	(By Construction)
AD = AD	(Common)
△BAD ≅ △CAD	(By SAS congruence rule)

22) Represent $\sqrt{3.5}$ on number line.

resentation of √3 .5 on the real number line:



Steps :

(i) Draw a line and mark A on it.

(ii) Mark a point B on the line drawn in step - (i) such that AB = 3. 5 units

(iii) Mark a point C on AB produced such that BC = 1unit

(iv) Find mid-point of AC. Let the midpoint be O

=> AC = AB + BC = 3.5 + 1 = 4.5

=> AO = OC = AC/2 = 4.5/2 = 2.25

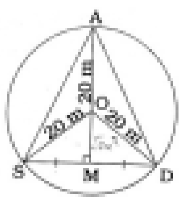
(v) Taking O as the center and OC = OA as radius drawn a semi-circle. Also draw a line passing through B perpendicular to OB. Suppose it cuts the semi-circle at D. Consider triangle OBD, it is right angled at B.

(vi) Taking B as the center and BD as radius draw an arc cutting OC produced at E. point E so obtained represents √3. 5 as BD = BE =√3.5 radius Thus, E represents the required point on the real number line.

Section D

- Give the answer in detailed. (Each 4 marks) [12]

23) A circular park of radius 20m is situated in a colony. Three boys Sanjay,Hitesh, and Ashish are sitting at equal distance on its boundary each having a toy phone in his hands to talk with each other. Find the length if the string of each phone.



Here, the circle with centre O represents the park and the points A, S and D represent the positions of Ankur, Syed and David respectively. Since Ankur, Syed and David are sitting at equal distances from the others, ΔASD is an equilateral triangle.

Then, drawing the perpendicular bisector of SD from its midpoint M, it will pass through O as well as A.

Suppose, SM = x m

$\therefore SD = 2SM = 2x$ m

Area of equilateral $\Delta ASD = \frac{\sqrt{3}}{4}(\text{side})^2$

$\therefore$ Area of equilateral $\Delta ASD = \frac{\sqrt{3}}{4} \times (2x)^2$

$\therefore$ Area of equilateral $\Delta ASD = \sqrt{3}x^2$ ... ...(1)

In ΔOMS, $\angle M = 90°$

$\therefore OM^2 = OS^2 - SM^2 = (20)^2 - (x)^2 = 400 - x^2$

$\therefore OM = \sqrt{400 - x^2}$

Now, area of $\Delta OSD = \frac{1}{2} \times SD \times OM$

$\therefore$ Area of $\Delta OSD = \frac{1}{2} \times 2x \times \sqrt{400 - x^2}$

$\therefore$ Area of $\Delta OSD = x\sqrt{400 - x^2}$ ... ...(2)

Here, ΔOAS, ΔOSD and ΔODA are congruent triangles.

Area of ΔASD = Area of ΔOAS + Area of ΔOSD + Area of ΔODA

$\therefore$ Area of $\Delta ASD = 3 \times$ Area of ΔOSD

$\therefore \sqrt{3} \cdot x^2 = 3 \times x\sqrt{400 - x^2}$

$\therefore x = \sqrt{3} \cdot \sqrt{400 - x^2}$

$\therefore x^2 = 3(400 - x^2)$

$\therefore x^2 = 1200 - 3x^2$

$\therefore 4x^2 = 1200$

$\therefore x^2 = 300$

$\therefore x = 10\sqrt{3}$

$SD = 2x = 2 \times 10\sqrt{3} = 20\sqrt{3}$ m

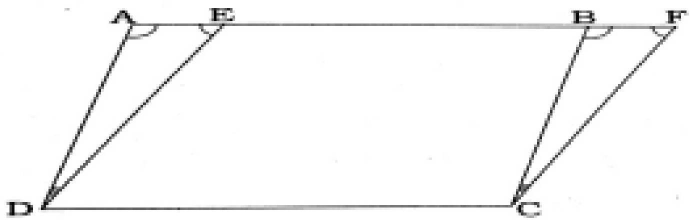
Thus, the length of the string of each phone is $20\sqrt{3}$ m.

24) Prove that parallelograms on the same base and between the same parallels are equal in area.

Theorem 9.1 : Parallelograms on the same base (or equal bases) and between the same parallels are equal in area.

Given : Parallelograms ABCD and EFCD are on the same base DC and between the parallels AF and DC.

To prove : ar (ABCD) = ar (EFCD).



Proof : In Δ ADE and Δ BCF,

$\angle$ EAD = $\angle$ FBC

(Corresponding angles formed by transversal AF of AD ∥ BC)

$\angle$ AED = $\angle$ BFC (Corresponding angles formed by transversal AF of DE ∥ CF)

AD = BC (Opposite sides of parallelogram ABCD)

$\therefore$ Δ ADE $\cong$ Δ BCF (AAS congruence rule)

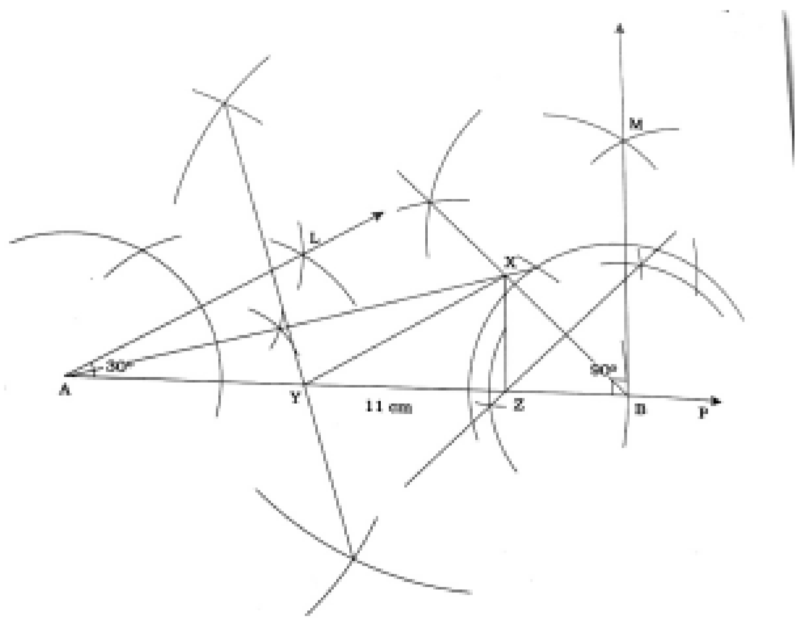
$\therefore$ ar (ADE) = ar (BCF)

Now, ar (ABCD) = ar (ADE) + ar (EDCB)

= ar (BCF) + ar (EDCB)

= ar (EFCD)

25) Construct a triangle XYZ in which Y= 30° , Z = 90° , and XY+YZ+ZX = 11cm.



Steps of construction :

(1) Draw any ray AP and from that obtain the line segment AB of length 11 cm.

(2) Construct ray AL such that ∠ LAB = 30°.

(3) Construct ray BM such that ∠ MBA = 90°.

(4) Draw the bisectors of ∠ LAB and ∠ MBA to intersect each other at X.

(5) Draw line segment XB. Draw the perpendicular bisector of XB to intersect AB at Z.

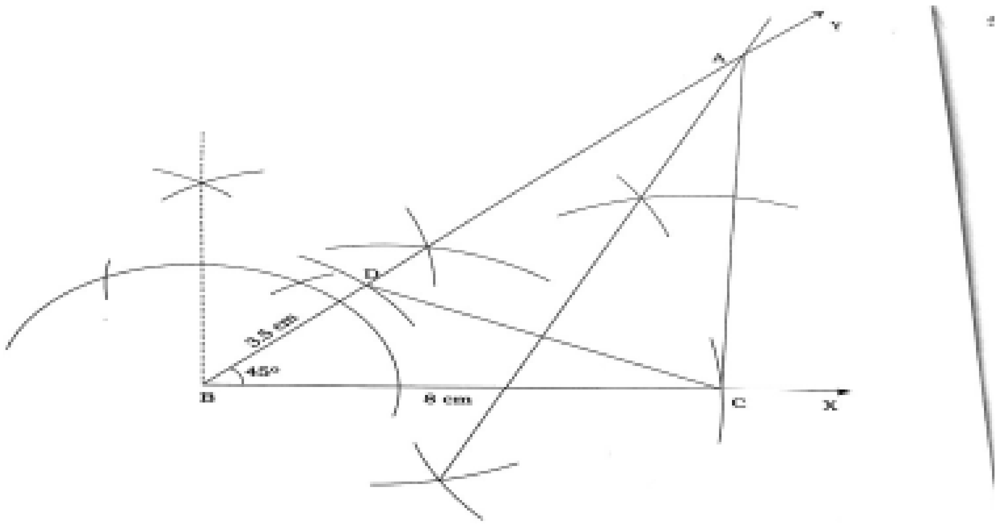
(6) Draw line segment XA. Draw the perpendicular bisector of XA to intersect AB at Y.

(7) Draw line segments XY and XZ.

Then, Δ XYZ is the required triangle.

OR

Construct a triangle ABC in which BC= 8cm, ∠B = 45 and AB-AC = 3.5cm.



Steps of construction :

(1) Draw any ray BX and from that obtain the line segment BC of length 8 cm.

(2) At B, draw ray BY such that ∠ YBC = 45°.

(3) With centre B and radius 3.5 cm, draw an arc to intersect ray BY at D.

(4) Draw line segment DC. Draw the perpendicular bisector of DC to intersect ray BY at

(5) Draw line segment AC.

Then, Δ ABC is the required triangle.

☻ ☻ ☻